

ପୃଥକ ସମୀକରଣ

Exercise - 2(a)

Solution

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1. ନିମ୍ନଲିଖିତ ଉକ୍ତି ଗୁଡ଼ିକରେ ଥିବା ତ୍ରୁଟିକୁ ସଂଶୋଧନ କରି ଲେଖ ।

(i) $x^2 - 4x + 4 = 0$ ସମୀକରଣର ମୂଳ ଦ୍ଵୟ ବାସ୍ତବ ଓ ଭିନ୍ନ ।

ମୂଳ ଦ୍ଵୟର ସାମୁଦ୍ର ଓ ଭିନ୍ନ 070

$$D = b^2 - 4ac = (-4)^2 - 4 \times 1 \times 4 = 16 - 16 = 0$$

$\therefore$ ମୂଳ ଦ୍ଵୟର ସାମୁଦ୍ର ଓ ସମାନ

(ii) $x^2 - 5x + 6 = 0$ ସମୀକରଣର ପ୍ରଭେଦକ 2 ଅଟେ ।

$$D = b^2 - 4ac = (-5)^2 - 4 \times 1 \times 6 = 25 - 24 = 1$$

$\therefore$ ପ୍ରଭେଦକ = 1

(iii) $ax^2 + bx - c = 0$ ସମୀକରଣର ମୂଳ ଦ୍ଵୟର ସମଷ୍ଟି $\frac{c}{a}$ ।

$ax^2 + bx + c = 0$ ସମୀକରଣର ମୂଳ ଦ୍ଵୟର ସମଷ୍ଟି $-b/a$

$\therefore$ ମୂଳ ଦ୍ଵୟର ସମଷ୍ଟି = $-\frac{b}{a}$

(iv) $ax^2 + bx + c = 0$ ସମୀକରଣର ମୂଳ ଦ୍ଵୟର ଗୁଣଫଳ $\frac{b}{a}$ ।

$\therefore$ ମୂଳ ଦ୍ଵୟର ଗୁଣଫଳ = c/a

(v) 1 ଓ -1 ମୂଳ ବିଶିଷ୍ଟ ଦ୍ଵିଘାତ ସମୀକରଣଟି $x^2 + 1 = 0$ ।

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \Rightarrow x^2 - (1 - 1)x + (1)(-1) = 0$$

$$\Rightarrow x^2 - 1 = 0$$

(vi) $x^2 = 0$ ଦ୍ଵିଘାତ ସମୀକରଣର ମୂଳ ସମାନ ନୁହେଁ ।

$x^2 = 0 \Rightarrow x = 0, 0$ $\therefore$ ମୂଳ ଦ୍ଵୟର ସମାନ

ସମସ୍ତ ସୂତ୍ର
 $ax^2 + bx + c = 0$
ଯାହାର ମୂଳଦ୍ଵୟ
 α ଓ β
ତତ୍ତ୍ଵାତ୍ମକ ଥାଆନ୍ତୁ
କିଛି ଲେଖା
ଲାଭୁଛି

(vii) $3x^2 - 2x - 1 = 0$ ସମୀକରଣର ମୂଳ ଦ୍ଵୟର ସମଷ୍ଟି $-\frac{3}{2}$ ।

$ax^2 + bx + c = 0$ ସମୀକରଣର ମୂଳ ଦ୍ଵୟର ସମଷ୍ଟି $= -\frac{b}{a}$

$b = -2$ & $a = 3$

ମୂଳ ଦ୍ଵୟର ସମଷ୍ଟି $= -(-2)/3 = 2/3$

(viii) $3x^2 - 2x - 1 = 0$ ସମୀକରଣର ମୂଳ ଦ୍ଵୟର ଗୁଣଫଳ $\frac{1}{3}$ ।

$ax^2 + bx + c = 0$ ସମୀକରଣର ମୂଳ ଦ୍ଵୟର ଗୁଣଫଳ $= c/a$

$c = -1$ & $a = 3$

$\frac{c}{a} = -\frac{1}{3}$ (Ans)

2. ନିମ୍ନଲିଖିତ ପ୍ରଶ୍ନମାନଙ୍କର ସଂକ୍ଷିପ୍ତ ଉତ୍ତର ଦିଅ ।

(i) ଗୋଟିଏ ଦ୍ଵିଘାତ ସମୀକରଣର ମୂଳଦ୍ଵୟ 3 ଓ -5 ହେଲେ, ସମୀକରଣଟି ନିରୂପଣ କର ।

ଦ୍ଵିଘାତ ସମୀକରଣ : $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$x^2 - (3 - 5)x + 3(-5) = 0$$

$$\Rightarrow \boxed{x^2 + 2x - 15 = 0} \quad (\text{Ans})$$

(ii) $mx^2 - 2x + (2m - 1) = 0$ ସମୀକରଣର ମୂଳଦ୍ଵୟର ଗୁଣଫଳ 3 ହେଲେ, m ର ମାନ ନିରୂପଣ କର ।

ମୂଳ ଦ୍ଵୟର ଗୁଣଫଳ = $\frac{c}{a}$ $c = 2m - 1$ & $a = m$

$$\Rightarrow \frac{2m - 1}{m} = 3 \Rightarrow 2m - 1 = 3m \Rightarrow \boxed{m = -1} \quad (\text{Ans})$$

(iii) $x^2 - px + 2 = 0$ ସମୀକରଣର ଗୋଟିଏ ମୂଳ 2 ହେଲେ, p ର ମାନ ନିରୂପଣ କର ।

Let's assume ମୂଳ ଦ୍ଵୟ : 2 ଓ p

$$x^2 - (2 + p)x + 2p = 0$$

$$\text{ମାତ୍ର } -(2 + p) = -p \Rightarrow 2 + p = p$$

$$\text{ତଥା } 2p = 2 \Rightarrow \boxed{p = 1}$$

$$\Rightarrow p = 2 + p = 2 + 1 = 3 \quad \boxed{p = 3} \quad (\text{Ans})$$

(iv) $4x^2 - 2x + c = 0$ ସମୀକରଣର ମୂଳଦ୍ଵୟ ଏକ ଓ ଅନ୍ୟଟି ହେଲେ, c ର ମାନ ନିରୂପଣ କର ।

ମୂଳ ଦ୍ଵୟ 0 ଓ ଅନ୍ୟଟି ହେଲେ $0 = 0$ $b^2 = 4ac$

$$(-2)^2 = 4 \times 4 \times c \Rightarrow 4 = 4 \times 4 \times c \Rightarrow \boxed{c = \frac{1}{4}} \quad (\text{Ans})$$

(v) $5x^2 + 2x + k = 0$ ସମୀକରଣର ଗୋଟିଏ ମୂଳ -2 ହେଲେ, k ର ମାନ ନିରୂପଣ କର ।

ମୂଳ ଦ୍ଵୟ α ଓ β ହେଲେ

ପ୍ରମାଣିତ ହେଉ $\alpha + \beta = -\frac{2}{5}$ ଓ $\alpha\beta = \frac{k}{5}$

Given $\alpha = -2 \Rightarrow -2 + \beta = -\frac{2}{5} \Rightarrow \beta = 2 - \frac{2}{5} = \frac{8}{5}$

Again $(-2)\beta = \frac{k}{5} \Rightarrow (-2)\frac{8}{5} = \frac{k}{5} \Rightarrow \boxed{k = -16}$ (Ans)

(vi) $x^2 - kx + 6 = 0$ ସମୀକରଣର ଗୋଟିଏ ମୂଳ 3 ହେଲେ, k ର ମାନ ନିରୂପଣ କର ।

Let's assume ମୂଳ ଦ୍ଵୟ α ଓ β

$\alpha + \beta = -(-k)$ $\alpha\beta = 6$

ଗୋଟିଏ ମୂଳ '3' $\Rightarrow 3\beta = 6 \Rightarrow \boxed{\beta = 2}$

$3 + 2 = k \Rightarrow \boxed{k = 5}$ (Ans) $(\because \alpha = 3)$

$$\begin{aligned} \alpha + \beta &= -\frac{b}{a} \\ \alpha\beta &= \frac{c}{a} \end{aligned}$$

(vii) $2x^2 + kx + 3 = 0$ ସମୀକରଣର ଦୁଇଟି ମୂଳ ବାସ୍ତବ ଓ ସମାନ ହେଲେ, k ର ମାନ ନିରୂପଣ କର ।

ମୂଳ ଦ୍ଵୟ ସମାନ $\Rightarrow D = 0 \Rightarrow b^2 = 4ac$

$k^2 = 4 \times 3 \times 2 \Rightarrow k^2 = 4 \times 6 \Rightarrow \boxed{k = 2\sqrt{6}}$ (Ans)

3. ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନ ପାଇଁ ଥିବା ସମ୍ଭାବ୍ୟ ଉତ୍ତରଗୁଡ଼ିକ ମଧ୍ୟରୁ ଠିକ୍ ଉତ୍ତରଟି ବାଛି ଲେଖ ।

(i) ନିମ୍ନଲିଖିତ ସମୀକରଣମାନଙ୍କ ମଧ୍ୟରୁ କେଉଁଟି x ରେ ଏକ ଦ୍ଵିଘାତ ସମୀକରଣ ?

(a) $x^2 - x - 12 = 0$

(b) $x^2 + \frac{1}{x^2} = 3$

(c) $x + \frac{3}{x} = x^2$

(d) $x(x-1)(x+5) = 0$

ଦ୍ଵିଘାତ ସମୀକରଣ : ଅଜ୍ଞିତ ରାଶିର ସର୍ବନିମ୍ନ ଘାତ = 2

(a) $x^2 - x - 12 = 0$ (a)

(b) $x^2 + \frac{1}{x^2} = 3$

$\Rightarrow x^4 + 1 = 3x^2 \Rightarrow x^4 - 3x^2 + 1 = 0$

(c) $x + \frac{3}{x} = x^2$

$\Rightarrow x^2 + 3 = x^3$

$\Rightarrow x^3 - x^2 + 3 = 0$

[ଘାତ : 3]

(d) $x(x-1)(x+5) = 0$

[ଘାତ : 3]

[ଘାତ : 4]

(ii) $7x^2 - 9x + 2 = 0$ ସମୀକରଣର ମୂଳଦ୍ଵୟର ସ୍ଵରୂପ କ'ଣ ?

(a) ବାସ୍ତବ ସଂଖ୍ୟା ଓ ପରସ୍ପରଠାରୁ ପୃଥକ୍ ।

(b) ବାସ୍ତବ ସଂଖ୍ୟା ଏବଂ ଏକ ଓ ଅନ୍ୟଟି ।

(c) ବାସ୍ତବ ହେବେ ନାହିଁ ।

(d) ଏଥିରୁ କେଉଁଟି ନୁହେଁ ।

ପ୍ରଭେଦକ : $D = b^2 - 4ac = (-9)^2 - 4 \times 7 \times 2$

$= 81 - 56 = 25$

ତେଣୁ $D > 0$ ଓ ଏହା ଠିକ୍

ମୂଳଦ୍ଵୟ ସଂଖ୍ୟା $\Rightarrow$ ମୂଳଦ୍ଵୟ ସାମ୍ୟ ଓ ଏହା ପୃଥକ୍ (a)
ପରିମେୟ

(iii) ନିମ୍ନଲିଖିତ ମଧ୍ୟରୁ କେଉଁଟି -6 ଓ 8 ମୂଳ ବିଶିଷ୍ଟ ଦ୍ଵିଘାତ ସମୀକରଣ ?

(a) $(x+6)(x+8)=0$

(b) $(x+6)(x-8)=0$

(c) $(x-6)(x+8)=0$

(d) $(x-6)(x-8)=0$

ଦ୍ଵିଘାତ ସମୀକରଣର ମୂଳ ଦ୍ଵୟ α ଓ β ହେଲେ
ଦ୍ଵିଘାତ ସମୀକରଣ : $(x-\alpha)(x-\beta)=0$

$\alpha = -6$ ଓ $\beta = 8$

ସମୀକରଣ $(x+6)(x-8)=0$ (b)

(iv) $3x^2 + 2\sqrt{5}x - 5 = 0$ ସମୀକରଣର ମୂଳଦ୍ଵୟ α ଓ β ହେଲେ $\alpha\beta$ ର ମୂଲ୍ୟ କେତେ ?

(a) 3

(b) $2\sqrt{5}$

(c) $\frac{2\sqrt{5}}{3}$

(d) $\frac{-5}{3}$

$\alpha\beta = \frac{c}{a} = \frac{-5}{3}$ (d)

(v) $4x^2 - 2x + \frac{1}{4} = 0$ ସମୀକରଣର ମୂଳଦ୍ଵୟ α ଓ β ହେଲେ $\alpha+\beta$ ର ମୂଲ୍ୟ କେତେ ?

(a) $\frac{1}{16}$

(b) 4

(c) $\frac{1}{2}$

(d) -8

$\alpha+\beta = \frac{-b}{a} = \frac{-(-2)}{4} = \frac{2}{4} = \frac{1}{2}$ (c)

(vi) $4x^2 + 3x + 7 = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β ହେଲେ $\frac{1}{\alpha} + \frac{1}{\beta}$ ର ମୂଲ୍ୟ କେତେ ?

- (a) $\frac{3}{7}$ (b) $-\frac{3}{7}$ (c) $\frac{7}{3}$ (d) $-\frac{7}{3}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{(-3)/4}{7/4} = -\frac{3}{7} \quad (b)$$

(vii) ଗୋଟିଏ ଦ୍ୱିଘାତ ସମୀକରଣର ମୂଳଦ୍ୱୟର ସମଷ୍ଟି ଓ ଗୁଣଫଳ ଯଥାକ୍ରମେ 4 ଓ $\frac{5}{2}$ ହେଲେ ସମୀକରଣଟି ନିମ୍ନଲିଖିତ ମଧ୍ୟରୁ କେଉଁଟି ?

(a) $2x^2 + 8x + 5 = 0$

(b) $2x^2 - 8x + 5 = 0$

(c) $2x^2 + 8x - 5 = 0$

(d) $2x^2 - 8x - 5 = 0$

$\alpha + \beta = 4$ $\alpha\beta = \frac{5}{2}$ ସମୀକରଣ $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$x^2 - 4x + \frac{5}{2} = 0 \Rightarrow 2x^2 - 8x + 5 = 0 \quad (b)$$

4. ନିମ୍ନଲିଖିତ ଦ୍ଵିଘାତ ସମୀକରଣମାନଙ୍କୁ ପୂର୍ଣ୍ଣ ବର୍ଗରେ ପରିଣତ କରି ସମାଧାନ କର ।

(i) $x^2 + x - 6 = 0$

$$\left[\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ ଉପାଦାନ} \right]$$

ଆମର ଉଷ୍ମ ସମାଧାନ କରିବୁ ।

$$x^2 + x - 6 = 0$$

$$\Rightarrow x^2 + x = 6 \quad [-c]$$

$$\Rightarrow 4 \times 1 (x^2 + x) = 4 \times 1 \times 6 \quad [-4ac]$$

$$\Rightarrow 4x^2 + 4x = 24$$

$$\Rightarrow (2x)^2 + 2 \cdot 2x + 1^2 = 1^2 + 24 \quad [b^2 - 4ac]$$

$$\Rightarrow (2x + 1)^2 = 24 + 1 = 25$$

$$\Rightarrow 2x + 1 = \sqrt{25} = \pm 5 \quad \sqrt{b^2 - 4ac}$$

$$\Rightarrow x = \frac{-1 \pm 5}{2} = \frac{-6}{2} \text{ (3) } \frac{4}{2} \Rightarrow \boxed{x = -3 \text{ (3) } 2}$$

(ii) $2x^2 - 9x + 4 = 0$

$$2x^2 - 9x + 4 = 0$$

$$\Rightarrow 2x^2 - 9x = -4$$

$$\Rightarrow 4 \times 2 [2x^2 - 9x] = 4 \times 2 (-4)$$

$$\Rightarrow 8 (2x^2 - 9x) = 8 (-4)$$

$$\Rightarrow 16x^2 - 72x = -32$$

$$\Rightarrow (4x)^2 - 2 \cdot 4x \cdot 9 + 9^2 = 9^2 - 32$$
$$= 81 - 32$$

$$\Rightarrow (4x - 9)^2 = 49$$

$$\Rightarrow 4x - 9 = \pm 7$$

$$\Rightarrow x = \frac{9 \pm 7}{4} = \frac{16}{4} \text{ or } \frac{2}{4}$$

$$\Rightarrow \boxed{x = 4 \text{ or } \frac{1}{2}} \quad (\text{Ans})$$

(iii) $14x^2 + x - 3 = 0$

$$14x^2 + x - 3 = 0$$

$$\Rightarrow 14x^2 + x = 3$$

$$\Rightarrow 4 \times 14 (14x^2 + x) = 4 \times 14 \times 3$$

$$\Rightarrow 4 \times 14 \times 14x^2 + 4 \times 14x = 168$$

$$\Rightarrow (28x)^2 + 2 \cdot 28x + 1^2 = 1^2 + 168$$

$$\Rightarrow (28x + 1)^2 = 169$$

$$\Rightarrow 28x + 1 = \pm 13$$

$$\Rightarrow x = \frac{-1 \pm 13}{28}$$

$$\Rightarrow x = \frac{-14}{28} \text{ or } \frac{12}{28}$$

$$\boxed{x = -\frac{1}{2} \text{ or } \frac{3}{7}}$$

$$(iv) 3x^2 - 32x + 12 = 0$$

$$\Rightarrow 3x^2 - 32x = -12$$

$$\Rightarrow 4 \times 3 (3x^2 - 32x) = 4 \times 3 (-12)$$

$$\Rightarrow 36x^2 - 12 \times 32x = 12 \times -12$$

$$\Rightarrow (6x)^2 - 2 \cdot 6x \cdot 32 + 32^2$$

$$= 32^2 - 12^2$$

$$\Rightarrow (6x - 32)^2 = (32 + 12)(32 - 12)$$

$$\Rightarrow (6x - 32)^2 = 44 \times 20 = 880$$

$$\Rightarrow 6x - 32 = \sqrt{880}$$

$$\Rightarrow x = \frac{32 \pm \sqrt{880}}{6}$$

$$= \frac{2}{6} [16 \pm \sqrt{220}]$$

$$\boxed{x = \frac{1}{3} [16 \pm \sqrt{220}]}$$

(Ans)

$$(v) x^2 + 2px - 3qx - 6pq = 0$$

$$\Rightarrow x^2 + x(2p - 3q) = 6pq$$

$$\Rightarrow 4 [x^2 + x(2p - 3q)] = 4 \times 6pq$$

$$\Rightarrow 4x^2 + 4x(2p - 3q) = 24pq$$

$$\Rightarrow (2x)^2 + 2 \cdot 2x(2p - 3q) + (2p - 3q)^2$$

$$= (2p - 3q)^2 + 24pq$$

$$\Rightarrow (2x + 2p - 3q)^2 = (2p + 3q)^2$$

$$[\because (a-b)^2 + 4ab = (a+b)^2]$$

$$\Rightarrow 2x + 2p - 3q = \pm (2p + 3q)$$

$$\Rightarrow 2x = (3q - 2p) \pm (2p + 3q)$$

$$= 3q - 2p + 2p + 3q \text{ or}$$

$$3q - 2p - 2p - 3q$$

$$\Rightarrow 2x = 6q \text{ or } -4p$$

$$\Rightarrow \boxed{x = 3q \text{ or } -2p} \quad (\text{Ans})$$

$$(vi) \sqrt{3}x^2 + 10x + 8\sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}x^2 + 10x = -8\sqrt{3}$$

$$\Rightarrow 4\sqrt{3}(\sqrt{3}x^2 + 10x) = 4\sqrt{3}(-8\sqrt{3})$$

$$\Rightarrow 4 \times 3x^2 + 40\sqrt{3}x = -96$$

$$\Rightarrow 12x^2 + 40\sqrt{3}x = -96$$

$$\Rightarrow (2\sqrt{3}x)^2 + 2 \cdot 2\sqrt{3}x \cdot 10 + 10^2 = 10^2 - 96$$

$$\Rightarrow (2\sqrt{3}x + 10)^2 = 4$$

$$\Rightarrow 2\sqrt{3}x + 10 = \pm 2$$

$$\Rightarrow x = \frac{-10 \pm 2}{2\sqrt{3}} = \frac{-12}{2\sqrt{3}} \text{ or } \frac{-8}{2\sqrt{3}}$$

$$\Rightarrow x = \frac{-6}{\sqrt{3}} \text{ or } \frac{-4}{\sqrt{3}} \\ = \frac{-6\sqrt{3}}{\sqrt{3}\sqrt{3}} \text{ or } \frac{-4\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$x = -2\sqrt{3} \text{ or } -\frac{4}{3}\sqrt{3}$$

(Ans)

$$(vii) 25x^2 + 30x + 7 = 0$$

$$\Rightarrow 25x^2 + 30x = -7$$

$$\Rightarrow (5x)^2 + 2 \cdot 5x \cdot 3 + 3^2 = 9 - 7$$

$$\Rightarrow (5x + 3)^2 = 2$$

$$\Rightarrow 5x + 3 = \pm\sqrt{2}$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{2}}{5} \quad (\text{Ans})$$

$$(viii) 3a^2x^2 + 8abx + 4b^2 = 0 \quad (a \neq 0)$$

$$\Rightarrow 3a^2x^2 + 8abx = -4b^2$$

$$\Rightarrow 4 \cdot 3a^2[3a^2x^2 + 8abx] = 4 \cdot 3a^2(-4b^2)$$

$$\Rightarrow 4 \times 9a^4x^2 + 4 \cdot 3a^2 \cdot 8abx \\ = -48a^2b^2$$

$$\Rightarrow (2 \cdot 3a^2x)^2 + 2 \cdot 2 \cdot 3a^2x \cdot 8ab + (8ab)^2$$

$$= (8ab)^2 - 48a^2b^2 \\ \Rightarrow (6a^2x + 8ab)^2 = 64a^2b^2 - 48a^2b^2$$

$$= 16a^2b^2 \\ \Rightarrow 6a^2x + 8ab = \pm 4ab$$

$$\Rightarrow x = \frac{-8ab \pm 4ab}{6a^2}$$

$$\Rightarrow x = \frac{-12ab}{6a^2} \text{ or } \frac{-4ab}{6a^2}$$

$$x = -2\frac{b}{a} \text{ or } -\frac{2}{3}\frac{b}{a}$$

(Ans)

$$(ix) x^2 + ax + b = 0$$

$$\Rightarrow x^2 + ax = -b$$

$$\Rightarrow 4(x^2 + ax) = -4b$$

$$\Rightarrow 4x^2 + 4ax = -4b$$

$$\Rightarrow (2x)^2 + 2 \cdot 2x \cdot a + a^2 = a^2 - 4b$$

$$\Rightarrow (2x + a)^2 = a^2 - 4b$$

$$\Rightarrow 2x + a = \pm \sqrt{a^2 - 4b}$$

$$\Rightarrow x = \frac{-a \pm \sqrt{a^2 - 4b}}{2} \quad (\text{Ans})$$

$$(x) x^2 + bx = a^2 - ab$$

$$\Rightarrow x^2 + bx = a^2 - ab$$

$$\Rightarrow 4(x^2 + bx) = 4(a^2 - ab)$$

$$\Rightarrow 4x^2 + 4bx = 4a^2 - 4ab$$

$$\Rightarrow (2x)^2 + 2 \cdot 2x + b^2 = b^2 + 4a^2 - 4ab$$

$$\Rightarrow (2x + b)^2 = (b - 2a)^2$$

$$\Rightarrow 2x + b = \pm (b - 2a)$$

$$\Rightarrow x = \frac{-b \pm (b - 2a)}{2}$$

$$= \frac{-b + b - 2a}{2} \quad \& \quad \frac{-b - b + 2a}{2}$$

$$= \frac{-2a}{2} \quad \& \quad \frac{-2b + 2a}{2}$$

$$\Rightarrow \boxed{x = -a \quad \& \quad (a - b)} \quad (\text{Ans})$$

5. ଦ୍ଵିଘାତ ସୂତ୍ର ପ୍ରଯୋଗ କରି ନିମ୍ନଲିଖିତ ସମୀକରଣମାନଙ୍କର ବୀଜ ବା ମୂଳ ନିରୂପଣ କର ।

(i) $4x^2 - 11x + 6 = 0$

$$a = 4 \quad b = -11 \quad c = 6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-11) \pm \sqrt{(-11)^2 - 4 \times 4 \times 6}}{2 \times 4}$$

$$= \frac{11 \pm \sqrt{121 - 96}}{8}$$

$$= \frac{11 \pm \sqrt{25}}{8} = \frac{11 \pm 5}{8}$$

$$x = \frac{16}{8} \text{ \& } \frac{6}{8}$$

$$x = 2 \text{ \& } \frac{3}{4} \quad (\text{Ans})$$

(ii) $(2x - 1)(x - 2) = 0$

$$2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow 2x^2 - 5x + 2 = 0$$

$$a = 2 \quad b = -5 \quad c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times 2}}{2 \times 2}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{4}$$

$$= \frac{5 \pm 3}{4} = \frac{5 \pm 3}{4}$$

$$x = \frac{8}{4} \text{ \& } \frac{2}{4} \quad x = 2 \text{ \& } \frac{1}{2} \quad (\text{Ans})$$

ALTERNATE

$$(2x - 1)(x - 2) = 0$$

$$\Rightarrow 2x - 1 = 0 \quad \text{or} \quad x - 2 = 0$$

$$\Rightarrow x = \frac{1}{2} \quad \text{or} \quad x = 2$$

$$(iii) x^2 - (1+\sqrt{2})x + \sqrt{2} = 0$$

$$x^2 - (1+\sqrt{2})x + \sqrt{2} = 0$$

$$a=1 \quad b = -(1+\sqrt{2}) \quad c = \sqrt{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-[-(1+\sqrt{2})] \pm \sqrt{(1+\sqrt{2})^2 - 4 \times 1 \times \sqrt{2}}}{2 \times 1}$$

$$x = \frac{(1+\sqrt{2}) \pm \sqrt{1+2+2\sqrt{2}-4\sqrt{2}}}{2}$$

$$= \frac{(1+\sqrt{2}) \pm \sqrt{1+2-2\sqrt{2}}}{2}$$

$$= \frac{(1+\sqrt{2}) \pm \sqrt{(1-\sqrt{2})^2}}{2}$$

$$= \frac{1+\sqrt{2} \pm 1-\sqrt{2}}{2}$$

$$= \frac{1+\sqrt{2}+1-\sqrt{2}}{2} / \frac{1+\sqrt{2}-1+\sqrt{2}}{2}$$

$$x = \frac{2}{2} \text{ \& } \frac{2\sqrt{2}}{2}$$

$$\boxed{x = 1 \text{ \& } \sqrt{2}} \quad (\text{Ans})$$

$$(iv) a(x^2+1) = x(a^2+1), \quad a \neq 0$$

$$a(x^2+1) = x(a^2+1)$$

$$\Rightarrow ax^2+a = x(a^2+1)$$

$$\Rightarrow ax^2 - (a^2+1)x + a = 0$$

$$x = \frac{-[-(a^2+1)] \pm \sqrt{(a^2+1)^2 - 4 \times a \times a}}{2 \times a}$$

$$= \frac{(a^2+1) \pm \sqrt{a^4+1+2a^2-4a^2}}{2a}$$

$$= \frac{(a^2+1) \pm \sqrt{a^4+1-2a^2}}{2a}$$

$$= \frac{(a^2+1) \pm \sqrt{(a^2-1)^2}}{2a}$$

$$= \frac{a^2+1 \pm (a^2-1)}{2a}$$

$$x = \frac{a^2+1+a^2-1}{2a}, \quad \frac{a^2+1-a^2+1}{2a}$$

$$= \frac{2a^2}{2a}, \quad \frac{2}{2a}$$

$$\boxed{x = a, \frac{1}{a}} \quad (\text{Ans})$$

$$(v) 6x^2 + 11x + 3 = 0$$

$$a = 6 \quad b = 11 \quad c = 3$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-11 \pm \sqrt{121 - 4 \cdot 6 \cdot 3}}{2 \cdot 6} \\ &= \frac{-11 \pm \sqrt{121 - 72}}{12} \\ &= \frac{-11 \pm \sqrt{49}}{12} = \frac{-11 \pm 7}{12} \end{aligned}$$

$$x = -\frac{18}{12}, -\frac{4}{12}$$

$$\boxed{x = -\frac{3}{2}, -\frac{1}{3}} \quad (\text{Ans})$$

$$(vi) 2x^2 + 41x - 115 = 0$$

$$a = 2 \quad b = 41 \quad c = -115$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-41 \pm \sqrt{41^2 - 4 \cdot 2 \cdot (-115)}}{2 \cdot 2} \end{aligned}$$

$$= \frac{-41 \pm \sqrt{1681 + 920}}{4}$$

$$= \frac{-41 \pm \sqrt{2601}}{4}$$

$$= \frac{-41 \pm 51}{4}$$

$$x = \frac{-41 - 51}{4}, \frac{-41 + 51}{4}$$

$$= -\frac{92}{4}, \frac{10}{4}$$

$$\boxed{x = -23, \frac{5}{2}} \quad (\text{Ans})$$

$$(vii) 12x^2 + x - 6 = 0$$

$$a = 12 \quad b = 1 \quad c = -6$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 12 \cdot (-6)}}{2 \cdot 12} \\ &= \frac{-1 \pm \sqrt{1 + 288}}{24} \\ &= \frac{-1 \pm \sqrt{289}}{24} = \frac{-1 \pm 17}{24} \end{aligned}$$

$$x = -\frac{18}{24}, \frac{16}{24}$$

$$x = -\frac{3}{4}, \frac{2}{3} \quad (\text{Ans})$$

$$(viii) (6x + 5)(x - 2) = 0$$

$$6x^2 - 12x + 5x - 10 = 0$$

$$\Rightarrow 6x^2 - 7x - 10 = 0$$

$$a = 6, \quad b = -7 \quad c = -10$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 6 \cdot (-10)}}{2 \cdot 6} \\ &= \frac{7 \pm \sqrt{49 + 240}}{12} \\ &= \frac{7 \pm \sqrt{289}}{12} = \frac{7 \pm 17}{12} \end{aligned}$$

$$x = \frac{24}{12}, -\frac{10}{12}$$

$$x = 2, -\frac{5}{6} \quad (\text{Ans})$$

$$(ix) 15x^2 - x - 28 = 0$$

$$a = 15 \quad b = -1 \quad c = -28$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 15 \cdot (-28)}}{2 \cdot 15}$$

$$= \frac{1 \pm \sqrt{1 + 1680}}{30}$$

$$= \frac{1 \pm \sqrt{1681}}{30} = \frac{1 \pm 41}{30}$$

$$x = \frac{1+41}{30}, \frac{1-41}{30}$$

$$= \frac{42}{30}, \frac{-40}{30}$$

$$x = \frac{7}{5}, -\frac{4}{3}$$

$$(x) (x+5)(x-5) = 39$$

$$x^2 - 5x + 5x - 25 = 39$$

$$\Rightarrow x^2 = 64 \Rightarrow x = \pm 8$$

6. ଯଦି $4x^2 - 13x + k = 0$ ସମୀକରଣର ଗୋଟିଏ ମୂଳ ଅପରଟିର 12 ଗୁଣ ହେଲେ k ର ମୂଲ୍ୟ ନିରୂପଣ କର ।

ମୂଳ ଦ୍ଵୟ α ଓ β ହେଲେ Given $\alpha = 12\beta$

$$\text{ମୂଳ ଦ୍ଵୟର ଯୋଗଫଳ} = -\frac{b}{a} = -\frac{(-13)}{4} = \frac{13}{4}$$

$$\Rightarrow \alpha + \beta = \frac{13}{4}$$

$$\Rightarrow 12\beta + \beta = \frac{13}{4} \Rightarrow 13\beta = \frac{13}{4} \Rightarrow \boxed{\beta = \frac{1}{4}}$$

$$\text{Then } \alpha = 12 \times \frac{1}{4} = 3$$

$$\text{ମୂଳ ଦ୍ଵୟର ଗୁଣଫଳ } \alpha\beta = \frac{k}{4} \Rightarrow 3 \times \frac{1}{4} = \frac{k}{4} \Rightarrow \boxed{k=3} \text{ (Ans)}$$

7. $x^2 - 5x + p = 0$ ସମୀକରଣର ଗୋଟିଏ ମୂଳ ଅପରଟି ଅପେକ୍ଷା 3 ଅଧିକ ହେଲେ p ର ମୂଲ୍ୟ ନିରୂପଣ କର ।

ସମୀକରଣର ମୂଳ ଦ୍ଵୟ α ଓ β ହେଲେ

$$\text{Given } \alpha = \beta + 3$$

$$\text{Then } \alpha + \beta = -\frac{b}{a} = -\frac{(-5)}{1} = 5$$

$$\Rightarrow \beta + 3 + \beta = 5 \Rightarrow 2\beta + 3 = 5 \Rightarrow 2\beta = 2 \Rightarrow \boxed{\beta = 1}$$

$$\text{Again } \alpha\beta = \frac{c}{a} = p$$

$$\Rightarrow 4 \cdot 1 = p \Rightarrow \boxed{p=4} \text{ (Ans)}$$

$$\boxed{\alpha = 4}$$

8. ଯଦି $2x^2 - 5x + 3 = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β ହୁଏ ତେବେ $\alpha^2\beta + \alpha\beta^2$ ର ମୂଲ୍ୟ ନିରୂପଣ କର ।

ସମୀକରଣର ମୂଳ ଦ୍ୱୟ α ଓ β

$$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$$

$$2\alpha^2 - 5\alpha + 3 = 0 \quad \text{ସମୀକରଣରେ} \quad \alpha + \beta = -\frac{b}{a} = -\frac{(-5)}{2} = \frac{5}{2}$$

$$\text{then } \alpha\beta(\alpha + \beta) = \frac{3}{2} \times \frac{5}{2} = \boxed{\frac{15}{4}} \text{ (Ans)} \quad \alpha\beta = \frac{c}{a} = \frac{3}{2}$$

9. ଯଦି $2x^2 - 6x + 3 = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β ହୁଏ ତେବେ $(\alpha+1)(\beta+1)$ ର ମୂଲ୍ୟ ନିରୂପଣ କର ।

$2x^2 - 6x + 3 = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β

$$\alpha + \beta = -\frac{b}{a} = -\frac{(-6)}{2} = 3 \quad \alpha\beta = \frac{c}{a} = \frac{3}{2}$$

$$(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$$

$$= \frac{3}{2} + 3 + 1 = 4 + \frac{3}{2} = \boxed{\frac{11}{2}} \text{ (Ans)}$$

10. ଯଦି $2x^2 - (p+1)x + p - 1 = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟର ଅଞ୍ଚର ଓ ଗୁଣଫଳ ସମାନ ହେଲେ p ର ମାନ ନିରୂପଣ କର ।

$$2x^2 - (p+1)x + (p-1) = 0 \text{ ସମୀକରଣର ମୂଳଦ୍ୱୟ } \alpha \text{ ଓ } \beta$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-(p+1)}{2} = \frac{p+1}{2} \quad \alpha\beta = \frac{c}{a} = \frac{p-1}{2}$$

$$\text{Given } \alpha - \beta = \alpha\beta \Rightarrow (\alpha - \beta)^2 = (\alpha\beta)^2$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\alpha\beta)^2 \Rightarrow \left(\frac{p+1}{2}\right)^2 - 4\left(\frac{p-1}{2}\right) = \left(\frac{p-1}{2}\right)^2$$

$$\Rightarrow \frac{(p+1)^2}{4} - 2(p-1) = \frac{(p-1)^2}{4} \Rightarrow \frac{(p+1)^2 - (p-1)^2}{4} = 2(p-1)$$

$$\Rightarrow \frac{4 \cdot p \cdot 1}{4} = 2(p-1) \Rightarrow p = 2p - 2 \Rightarrow \boxed{p = 2} \text{ (Ans)}$$

11. ଯଦି $5x^2 - 3x - 2 = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β ହୁଏ, ତେବେ ପ୍ରମାଣ କର ଯେ $\alpha^3 + \beta^3 = \frac{117}{125}$

$$5x^2 - 3x - 2 = 0 \text{ ସମୀକରଣର ମୂଳଦ୍ୱୟ } \alpha \text{ ଓ } \beta \text{ ହେଲେ}$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{5} = \frac{3}{5} \quad \alpha\beta = \frac{c}{a} = -\frac{2}{5}$$

we know

$$(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$$

$$\left(\frac{3}{5}\right)^3 = \alpha^3 + \beta^3 + 3\left(-\frac{2}{5}\right)\left(\frac{3}{5}\right) \Rightarrow \frac{27}{125} = \alpha^3 + \beta^3 - \frac{18}{25}$$

$$\Rightarrow \alpha^3 + \beta^3 = \frac{27}{125} + \frac{18}{25} = \frac{27 + 90}{125} = \frac{117}{125} = \text{RHS (Proved)}$$

12. ଯଦି $5x^2 + 17x + 6 = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β ହୁଏ ତେବେ $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ ର ମୂଲ୍ୟ ନିରୂପଣ କର ।

$$5x^2 + 17x + 6 = 0 \quad \text{ସମୀକରଣର ମୂଳଦ୍ୱୟ } \alpha \text{ ଓ } \beta \text{ ହେଲେ}$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{17}{5} \quad \alpha\beta = \frac{c}{a} = \frac{6}{5}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\left(-\frac{17}{5}\right)^2 - 2\left(\frac{6}{5}\right)}{\left(\frac{6}{5}\right)^2}$$

$$= \left[\frac{289}{25} - \frac{12}{5} \right] \times \frac{25}{36} = \frac{289 - 60}{25} \times \frac{25}{36} = \boxed{\frac{229}{36}} \quad (\text{Ans})$$

13. $x^2 - 8x + 16p = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β ହେଲେ $\frac{\alpha\beta}{\alpha + \beta}$ ପରିପ୍ରକାଶକୁ p ମାଧ୍ୟମରେ ପ୍ରକାଶ କର ।

$$x^2 - 8x + 16p = 0 \quad \text{ସମୀକରଣର ମୂଳଦ୍ୱୟ } \alpha \text{ ଓ } \beta$$

$$\alpha + \beta = -\frac{b}{a} = -(-8) = 8 \quad \alpha\beta = \frac{c}{a} = 16p$$

$$\frac{\alpha\beta}{\alpha + \beta} = \frac{16p}{8} = \boxed{2p} \quad (\text{Ans})$$

14. ଯଦି $x^2 - 2(5+2m)x + 3(7+10m) = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ ଏକ ଓ ଅଭିନ୍ନ ହୁଏ, m ର ମୂଲ୍ୟ ନିରୂପଣ କର ।

$$x^2 - 2(5+2m)x + 3(7+10m) = 0$$

ସମୀକରଣର ମୂଳଦ୍ୱୟ ଏକ ଓ ଅଭିନ୍ନ $\Rightarrow D = 0$

$$\Rightarrow b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$$

$$\Rightarrow [-2(5+2m)]^2 = 4 \times 1 \times 3(7+10m)$$

$$\Rightarrow 4(25+4m^2+20m) = 4 \cdot 3(7+10m)$$

$$\Rightarrow 25+4m^2+20m = 21+30m$$

$$\Rightarrow 4m^2-10m+4=0 \Rightarrow 2m^2-5m+2=0$$

$$\Rightarrow m = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 2}}{2 \cdot 2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$$

$$\Rightarrow m = \frac{8}{4}, \frac{2}{4} \Rightarrow \boxed{m = 2, \frac{1}{2}} \text{ (Ans)}$$

15. (i) ଯଦି $a = b = c$ ହୁଏ, ତେବେ ପ୍ରମାଣ କର ଯେ ସମୀକରଣ

$$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0 \text{ ର ମୂଳଦ୍ୱୟ ବାସ୍ତବ ଏବଂ ଏକ ଓ ଅଭିନ୍ନ ।}$$

$$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$$

$$3(x-a)(x-a) = 0 \quad (\because a=b=c)$$

$$\Rightarrow 3(x-a)^2 = 0 \Rightarrow (x-a)^2 = 0 \Rightarrow x-a = 0 \Rightarrow \boxed{x=a} \text{ (proved)}$$

(ii) ଯଦି $a+b+c=0$ ଏବଂ $a, b, c \in \mathbb{Q}$ ହୁଏ, ତେବେ ପ୍ରମାଣ କର ଯେ

$$(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0 \text{ ସମୀକରଣର ବୀଜଦ୍ୱୟ ପରିମେୟ ହେବେ ।}$$

$$\text{Given } a+b+c=0$$

$$(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0$$

$$\Rightarrow (-a-a)x^2 + (-b-b)x + (-c-c) = 0 \quad \left[\begin{array}{l} \because a+b+c=0 \\ b+c=-a \\ c+a=-b \\ a+b=-c \end{array} \right]$$

$$\Rightarrow (-2a)x^2 + (-2b)x + (-2c) = 0$$

$$\Rightarrow ax^2 + bx + c = 0$$

$$D = b^2 - 4ac = [-(a+c)]^2 - 4ac = (a+c)^2 - 4ac$$

$$= (a-c)^2$$

$\therefore D$ ଏକ ପୂର୍ଣ୍ଣ ବର୍ଗ ସଂଖ୍ୟା

$\Rightarrow$ ସମୀକରଣର ମୂଳ ଦ୍ୱୟ ପରିମେୟ [Proved]

16. ଗୋଟିଏ ଦ୍ଵିଘାତ ସମୀକରଣର ମୂଳଦ୍ଵୟର ସମଷ୍ଟି 3 ଓ ମୂଳଦ୍ଵୟର ବର୍ଗର ସମଷ୍ଟି 29 ହେଲେ, ସମୀକରଣଟି ନିରୂପଣ କର ।

ଦ୍ଵିଘାତ ସମୀକରଣର ମୂଳଦ୍ଵୟ α ଓ β ହେଲେ

Given $\alpha + \beta = 3$ & $\alpha^2 + \beta^2 = 29$

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta \Rightarrow 3^2 = 29 + 2\alpha\beta$$

$$\Rightarrow 9 = 29 + 2\alpha\beta \Rightarrow 2\alpha\beta = -20 \Rightarrow \underline{\underline{\alpha\beta = -10}}$$

ଦ୍ଵିଘାତ ସମୀକରଣ

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\boxed{x^2 - 3x - 10 = 0} \quad (\text{Ans})$$

17. ଯଦି $2x^2 - 4x + 2 = 0$ ସମୀକରଣର ମୂଳଦ୍ଵୟ α ଓ β ହେଲେ ପ୍ରମାଣ କର ଯେ

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 4\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 2\alpha\beta = 12$$

$2x^2 - 4x + 2 = 0$ ସମୀକରଣର ମୂଳଦ୍ଵୟ α ଓ β

$$\alpha + \beta = -\frac{b}{a} = -\frac{(-4)}{2} = \frac{4}{2} = 2 \quad \alpha\beta = \frac{c}{a} = \frac{2}{2} = 1$$

LHS:

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 4\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 2\alpha\beta$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 4\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 2\alpha\beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 4\frac{\alpha + \beta}{\alpha\beta} + 2\alpha\beta$$

$$= \frac{2^2 - 2 \cdot 1}{1} + 4 \cdot \frac{2}{1} + 2 \cdot 1 = 4 - 2 + 8 + 2 = 12 = \text{RHS (Proved)}$$

18.(i) ଯଦି $ax^2 + bx + c = 0$ ସମୀକରଣର ଗୋଟିଏ ମୂଳ ଅପରଟିର 4 ଗୁଣ ହୁଏ, ତେବେ ପ୍ରମାଣ କର ଯେ $4b^2 = 25ac$ ।

$ax^2 + bx + c = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β

Given $\alpha = 4\beta$

$$\alpha + \beta = -\frac{b}{a} \Rightarrow 4\beta + \beta = -\frac{b}{a} \Rightarrow 5\beta = -\frac{b}{a} \Rightarrow \beta = -\frac{b}{5a}$$

$$\alpha\beta = \frac{c}{a} \Rightarrow 4\beta \cdot \beta = \frac{c}{a} \Rightarrow 4\beta^2 = \frac{c}{a}$$

$$\Rightarrow 4\left(-\frac{b}{5a}\right)^2 = \frac{c}{a} \Rightarrow 4\frac{b^2}{25a^2} = \frac{c}{a} \Rightarrow \boxed{4b^2 = 25ac} \text{ (proved)}$$

(ii) ଯଦି $x^2 - px + q = 0$ ସମୀକରଣର ଗୋଟିଏ ମୂଳ ଅପରଟିର 2 ଗୁଣ ହୁଏ, ତେବେ ପ୍ରମାଣ କର ଯେ $2p^2 = 9q$ ।

$x^2 - px + q = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ α ଓ β

Given $\alpha = 2\beta$

$$\alpha + \beta = -(-p) = p \Rightarrow 2\beta + \beta = p \Rightarrow 3\beta = p \Rightarrow \beta = p/3$$

$$\alpha\beta = q \Rightarrow 2\beta \cdot \beta = q \Rightarrow 2\beta^2 = q \Rightarrow 2\left(\frac{p}{3}\right)^2 = q$$

$$\Rightarrow 2\frac{p^2}{9} = q \Rightarrow \boxed{2p^2 = 9q} \text{ (proved)}$$

19.(i) ଯଦି $41x^2 - 2(5a+4b)x + (a^2+b^2) = 0$ ସମୀକରଣର ମୂଳଦ୍ୱୟ ସମାନ ହୁଅନ୍ତି, ତେବେ ପ୍ରମାଣ କର

$$\text{ଯେ, } \frac{a}{b} = \frac{5}{4}$$

$$41x^2 - 2(5a+4b)x + (a^2+b^2) = 0$$

$$\text{ସମୀକରଣର ମୂଳଦ୍ୱୟ ସମାନ ହେଲେ } D=0 \quad b^2 = 4ac$$

$$\Rightarrow [-2(5a+4b)]^2 = 4 \times 41 \times (a^2+b^2)$$

$$\Rightarrow 4(5a+4b)^2 = 4 \cdot 41 \cdot (a^2+b^2)$$

$$\Rightarrow 25a^2 + 16b^2 + 40ab = 41a^2 + 41b^2$$

$$\Rightarrow 16a^2 + 25b^2 - 40ab = 0 \Rightarrow (4a)^2 + (5b)^2 - 2 \cdot 4a \cdot 5b = 0$$

$$\Rightarrow (4a - 5b)^2 = 0 \Rightarrow 4a = 5b \Rightarrow \boxed{\frac{a}{b} = \frac{5}{4}} \quad (\text{Ans})$$

(ii) ଯଦି $x^2 + px + q = 0$ ସମୀକରଣର ବୀଜଦ୍ୱୟର ସମଷ୍ଟି ସେମାନଙ୍କର ବର୍ଗର ସମଷ୍ଟି ସହ ସମାନ ହୁଏ, ତେବେ ଦର୍ଶାଅ ଯେ, $2q = p(p+1)$

$$\text{ସମୀକରଣର ମୂଳଦ୍ୱୟ } \alpha \text{ ଓ } \beta$$

$$\text{Given } \alpha + \beta = \alpha^2 + \beta^2$$

$$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$$

$$x^2 + px + q = 0 \quad \text{ସମୀକରଣରେ } \alpha + \beta = -p \quad \alpha\beta = q$$

$$-p = (-p)^2 - 2q$$

$$\Rightarrow -p = p^2 - 2q \Rightarrow p^2 + p = 2q \Rightarrow \boxed{p(p+1) = 2q} \quad (\text{Ans})$$

(iii) ଯଦି $x^2 + px + q = 0$ ସମୀକରଣର ଗୋଟିଏ ବାଜ ଅନ୍ୟଟିର ବର୍ଗ ହୁଏ, ତେବେ ଦର୍ଶାଅ ଯେ,
 $p^3 + q^2 + q = 3pq$

$x^2 + px + q = 0$ ସମୀକରଣର ବାଜମୂଳ α ଓ β ହେଲେ
Given $\alpha = \beta^2$

$$\alpha\beta = q \Rightarrow \beta^2 \cdot \beta = q \Rightarrow \beta^3 = q$$

$$\alpha + \beta = -p \Rightarrow \beta^2 + \beta = -p \Rightarrow p = -(\beta^2 + \beta)$$

LHS

$$p^3 + q^2 + q = [-(\beta^2 + \beta)]^3 + (\beta^3)^2 + \beta^3$$

$$= (-1) [\beta^6 + \beta^3 + 3\beta^2\beta(\beta^2 + \beta)] + \beta^6 + \beta^3$$

$$= -\beta^6 - \beta^3 - 3\beta^3(\beta^2 + \beta) + \beta^6 + \beta^3$$

$$= -3\beta^3(\beta^2 + \beta)$$

RHS $3pq = 3 \cdot [-(\beta^2 + \beta)]\beta^3 = -3\beta^3(\beta^2 + \beta)$

$$\text{LHS} = \text{RHS} \text{ [Proved]}$$

20. ଯଦି $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ ସମୀକରଣର ବୀଜଦ୍ୱୟ ସମାନ ହୁଏ ତେବେ ଦର୍ଶାଅ ଯେ,

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$$

ସମୀକରଣର ବୀଜଦ୍ୱୟ ସମାନ ହେଲେ

$$\alpha + \beta = -\frac{b(c-a)}{a(b-c)} \Rightarrow \alpha + \alpha = -\frac{b}{a} \frac{(c-a)}{(b-c)}$$

$$\Rightarrow 2\alpha = -\frac{b}{a} \frac{(c-a)}{(b-c)} \Rightarrow \alpha = -\frac{b}{2a} \frac{(c-a)}{(b-c)}$$

$$\alpha\beta = \frac{c(a-b)}{a(b-c)} \Rightarrow \alpha^2 = \frac{c}{a} \frac{(a-b)}{(b-c)}$$

$$\Rightarrow \left(-\frac{b}{2a}\right)^2 \left[\frac{(c-a)}{(b-c)}\right]^2 = \frac{c}{a} \frac{(a-b)}{(b-c)}$$

$$\Rightarrow \frac{b^2}{4a^2} \frac{(c-a)^2}{(b-c)^2} = \frac{c}{a} \frac{(a-b)}{(b-c)} \Rightarrow \frac{b^2}{4a} \frac{(c-a)^2}{(b-c)} = c(a-b)$$

$$\Rightarrow b^2(c^2 + a^2 - 2ac) = 4ac(a-b)(b-c)$$

$$\Rightarrow b^2c^2 + a^2b^2 - 2ab^2c = 4ac(ab - ac - b^2 - bc)$$

$$\Rightarrow b^2c^2 + a^2b^2 - 2ab^2c = 4a^2bc - 4a^2c^2 - 4ab^2c - 4abc^2$$

$$\Rightarrow a^2b^2 + b^2c^2 + 4a^2c^2 + 2ab^2c - 4a^2bc + 4abc^2 = 0$$

$$\Rightarrow (ab + bc - 2ac)^2 = 0 \quad \left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \right]$$

$$\Rightarrow ab + bc = 2ac$$

$$\Rightarrow \frac{ab}{abc} + \frac{bc}{abc} = \frac{2ac}{abc} \Rightarrow \boxed{\frac{2}{b} = \frac{1}{c} + \frac{1}{a}} \quad (\text{proved})$$